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PAPER NO: 1

EXCHANGE RATE REGIME AND A HOUSEHOLD’S CHOICE OF DEBT

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Summary

The present paper looks at the impact of exchange rate regime and a household's choice of debt. One of the characteristics of economic transition in eastern European countries has been an increase in overall debt holding. Standard economic theory assumes the relationship S=I. According to this relationship, households should use debt only to purchase durable goods; however, in some eastern European countries, there has been a large increase in consumer loans, economic behavior not recognized under the standard no-Ponzi scheme assumption of economic models. This paper aims to investigate the case when debt is used to live above a household's standard budget constraint. Our model shows that the significant impact on the choice of the amount the debt households are willing to hold is due to the choice of the exchange rate regime made by the central bank. The model investigates a household's behavior in two main cases: a stable exchange rate regime (exchange rate regime with FX risk) and a variable exchange rate regime (exchange rate regime without exchange rate risk). Households make different choices under alternative exchange rate regimes; this pattern of behavior is presented in the model and verified by the data.

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1. Introduction

The two most basic assumptions used in macroeconomic models are that savings equal loans and that households are not allowed to break their budget constraint. This paper eliminates these two assumptions with the main objective of investigating how households behave under alternative exchange rate regimes[1] when these two assumptions are not used.

The standard approach to economic modeling is to use the two stated economic assumptions in a general equilibrium model and then analyze a particular behavior under certain shocks. This paper takes the alternative route. We remove the two assumptions and investigate the behavior of households under two different exchange rate regimes. The main implication of the paper lies in the assumption that the behavior of economic participants will change when there is a change in the exchange rate regime.

Before we move any further, we have to define an exchange rate regime for the purposes of this paper. One of the basic definitions of exchange rate regimes was provided by the IMF and this can be found in Von Hagen and Zhou (2002), Frankel (1999) and Crockett (2003). These authors have three main exchange rate regimes: fixed, intermediate and free floating. However, the definition of these regimes is based on the amount of movement in the exchange rate, not on the direction of the exchange rate. In this paper, we will assume alternative definitions of the exchange rate regime.

Definition: under a stable exchange rate regime, the central bank keeps the exchange rate fixed at a value or close to one central value. Over time, the exchange rate does not exhibit a clear directional movement; the movement in the exchange rate is similar to a flat line or mean reverting series[2].

Definition: under a variable exchange rate regime, the central bank actively changes the exchange rate and uses monetary policy to create a clear directional movement in the exchange rate[3].

The definitions of exchange rate regimes as we have presented them are based on the direction of the exchange rate regime, not on the volatility of the exchange rate regime.

Does a **choice** of the exchange rate regime, as we have defined it, have an important impact on the behavior of economic participants? This paper will argue that it does. We will investigate how a choice of the exchange rate regime, fixed vs. stable, based on our definition, affects the behavior of economic participants. The full scope of the possible effects is large and it was fully developed in Vidaković (2013). In this paper, we will focus on the choice of the amount of credit households are willing to hold under each exchange rate regime.

The actual choice of the exchange rate regime will at the same time determine the behavior of economic participants. In essence, we are dealing with multiple model agents. Economic participants have one model for each state of the system (choice of the exchange rate regime). Since the model for each exchange rate regime is different, the behavior of economic participants will be different as well.

One of the best examples to investigate economic switching in a standard economic model can be found in Farmer, Waggoner and Zha (2007), who investigate new Keynesian models and regime switching. The same authors also investigate what happens when there are forward switching expectations in Farmer, Waggoner and Zha (2009).

[1] The author would like to thank prof. dr. sc. Franjo Štiblar, prof. dr. sc. Žan Jan Oplotnik and Domagoj Tolić for help with computer programming.

[2] The views expressed herein are those of the author and do not necessarily reflect the views of anyone else.

[3] There is also a third possibility of a free floating regime, where the central bank does not participate in the FX market; however, this regime is of no interest to us.

Others investigate what happens when agents have multiple models, such as Cogley and Sargent (2008). The issue of economic switching and multiple models in the economy has been investigated in several papers, but it has not yet been used to investigate how a change in the exchange rate affects the choice of credit.

The rest of the paper is separated in the following way. Part two will develop a model with two states of the system: stable and variable exchange rate regimes. Part three will mathematically simulate the model and determine the household's preferences in terms of debt under alternative exchange rate regimes. Part four will test the model on empirical data. Part five concludes.

2. The model

In this part of the paper, we will create the economic model where households are allowed to have debt and increase their debt holding. The second economic participant is banks. Our focus will be on the different choices under alternative exchange rate regimes.

2.1. Households

We will assume households have two different models, one for each state of the monetary policy regimes. Our focus will be on the amount of credit households are willing to have under each exchange rate regime choice. Households know the state of monetary policy. They do not assume or expect a switch in monetary policy. Monetary policy is given exogenously to households by the central bank and we assume the credibility of the central bank.

2.1.1. Households under a stable exchange rate regime

Households are utility maximizing agents, where the utility comes from consumption. The time is discrete, t=0,1,2,3…. The object of households is to maximize consumption over an infinite period of time. Households make their expectations under the rational expectations hypothesis and they do not assume there will be an exchange rate regime switch. The maximization objective of the households is:

$$\max \left[\sum_{t=0}^{\infty} \beta^{t} u(c_t) \right] \tag{1}$$

The u(c) function is a continuous, twice differentiable function, and β is a discount factor.

In this model, we will use a novel approach by **separating savings from credit** by focusing on the household's cash flow. This approach will allow households to have changes in both credit and savings at the same time. The assumptions from standard textbooks (Blanchard and Fisher 1989, page 69), where savings equal loans and there is no Ponzi scheme, are removed. We will investigate a case where households can save and increase debt in parallel, since this case can be found in the data. Credit is obtained from the bank. It is possible for the household to have payments both for savings and for credit repayments in the same time period. The inflow of money for households in any time period is:

$$I_t = w^e_t + \tau S_{t-1} + \phi_t \qquad (2)$$

The inflow of money I comes from wage w, new debt ϕ and the portion of liquidated savings τ, where the values of τ are 0< τ <1. S_{t-1} is the total savings the household has accumulated up to time period t. In this model, the only risk for the household comes from wages. Wage w follows a simple autoregressive process: $w^e_t = \alpha w_{t-1} + \varepsilon$, where α is the autoregressive coefficient and ε is the disturbance with distribution N(0,σ).
Following the inflow equation, we can also present the expenditure equation at time t. The expenditure equation is the outflow for households, which we define as:

$$\mathrm{E}_t = c_t + s_t + \kappa \Phi_{t-1} \qquad (3)$$

Household expenditures or outflows E can be divided into consumption c, savings s and the portion of the existing debt paid off in that time period. Total household debt is Φ and the portion of debt paid off in time period t is κ, where 0<κ<1; s_t is the new savings in time period t.
The inflow and expenditure equation show us the household can borrow, repay debt, liquidate savings and create new savings all in the same time period. We also allow households to repay a portion of existing debt and to obtain new debt in the same time period, just like it can liquidate savings and create new savings at the same time.
Savings and debt accumulate over time and this accumulation can be expressed by the two following equations:

$$\Phi_t = \sum_{t=0}^{t-1} \phi_{i-1} (1 + r_i^*)^{i-1} \text{ with some maximum value of } \Phi^* \qquad (4)$$

$$S_t = \sum_{t=0}^{t-1} s_{i-1} (1 + r_i)^{i-1} \qquad (5)$$

The cost of debt for households is r*; this is the rate the bank is offering to the household. We assume the interest rate is the same for each household, exogenous and perfectly inelastic for any level of demand. The household also gets a savings rate of r from the bank.
The equilibrium is E=I, so now we can solve equations (2) and (3) for c and get:

$$c_t = w^e_t + \tau S_{t-1} + \phi_t - s_t - \kappa \Phi_{t-1} \qquad (6)$$

Equation (6) represents the flow of consumption in every time period for households. As equation (6) shows, there is a possibility for households to decrease total savings, have new savings, get new debt and repay old debt in one time period. The consumption equation (6) is the transition equation in our model.
The main characteristic of the model and the difference between this model and standard models is the combination of debt and savings at the same time. With the inclusion of new debt ϕ in the consumption equation (2), we have created a possibility for households to have the **desired** level of consumption, which removes the standard budget

constraint. Therefore, the choice of debt will also depend on the lifestyle the household wants to live, under the assumption the bank is willing to extend credit.

By using the model set up thus far, we will now set up the Bellman equation for households under a stable exchange rate regime. The value function is:

$$V(A) = \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{7}$$

The terminal condition will be the transversality condition. The control function for the problem is u(c), and so the Bellman equation is:

$$V(A_t) = \max_c \left[\frac{c^{1-\theta} - 1}{(1-\theta)} + \beta V(A_{t+1}) \right] \tag{8}$$

Subject to equation (6). Out of the recursion presented in the Bellman equation, households will obtain a policy function h(c*,ϕ*,s*), and they will plan their optimal path of consumption over time, where * represents the optimal choice as calculated by using the Bellman equation.

2.1.2. Model of a household's behavior under variable exchange rate policies

In the previous part, the main risk for households came from changes in wages. Now, we will expand the model and introduce the FX risk. If we have a variable exchange rate, banks will have to hedge the currency risk and the best way to do that is to transfer the risk to the customer. Risk can be easily transferred by lending in a foreign currency or embedding a foreign currency clause in loans. Because of this transference of the FX risk by banks, under a variable exchange rate households will take on the exchange rate risk every time they get a loan from the bank. In order to hedge their own position, households will have to save in a foreign currency.

Under a variable exchange rate, with the foreign currency clause embedded in their loans, households do not know what their amount of debt will be in each period in the local currency. The income function with FX risk will be:

$$I_t^e = w_t^e + \tau S_{t-1}^e + \phi_t \tag{9}$$

Wage w and savings are stochastic with the same properties as in the model with a fixed exchange rate. The new debt is not stochastic since households know the exchange rate at time t. The expenditure function has to be augmented:

$$\mathrm{E}_t^e = c_t^e + s_t + \kappa \Phi_{t-1}^e \tag{10}$$

Consumption is expected and so is the amount of debt the household will have in the current time period. The actual amount of the debt repayment in the local currency is not fixed since the debt is denominated in the foreign currency and changes in the exchange rate will cause changes in the amount of debt the household has. The value function is:

$$V(A) = \max E\left[\sum_{t=0}^{n} \beta^i u(c_i)\right] \qquad (11)$$

By equating I^e and E^e, we get the following function for consumption:

$$c_t^e = w_t^e + \tau S_{t-1}^e + \phi_t - s_t - \kappa \Phi_{t-1}^e \qquad (12)$$

where the control function is defined as:

$$f(\tau, s, \phi) \qquad (13)$$

The Bellman equation is:

$$V(\tau, s, \phi) = \max_{c,\phi}\left[f(\tau, s, \phi) - E\left[\sum_{n-t}^{n} \beta^i u(c_i)\right]\right] \qquad (14)$$

From this recursion, households form the policy function h(τ*,s*,ϕ*). Here, the values τ*, s* and ϕ* are optimal values to solve the Bellman equation. We can also present the solution to the Bellman equation (14) as the values of total debt and savings households are willing to have in any time period h(S*,Φ*). Under a variable exchange rate, the household does not choose how much new debt to obtain, but rather chooses the **total amount** of debt it is willing to hold.

For each exchange rate regime (state of the system), households have different policy functions. In the model with a fixed exchange rate regime, households did not care about the level of debt because their consumption was not affected by changes in the exchange rate. In the model with a variable exchange rate regime, households take into account the total amount of debt they have since the size of the debt and debt repayments are stochastic. This is directly imposed by the foreign exchange rate risk created by the variable exchange rate regime chosen by the central bank.

We could look at the whole regime choice from a different perspective as well. By using the logic presented in Santini (2007) and Vidaković (2008), we could look at the choice of monetary policy as an implied budget constraint. Under the stable exchange rate regime, households have a soft budget constraint because they can borrow as much as banks are willing to lend them. Under the variable exchange rate regime, households have a hard budget constraint. If households want to have more debt, they have to take on the exchange risk.

From the model, it is clear the exchange rate can be an important policy tool to control the level of debt households are willing to hold, thus making the exchange rate a tool for controlling credit policy in the economy. We have directly seen in the model how the choice of monetary policy affects the behavior of households.

The stable exchange rate regime in essence gives the household freedom when it comes to debt. In that case, the only determinant of the level of debt households will have is their time preference of consumption and the debt limit imposed by the bank. In case households want to have a preference towards present consumption, they will obtain as much debt as possible and consume as much as they can in the near future. If the individual household preference is towards the future, then households will increase their savings rate and save in order to be able to consume more in the future.

Given the model presented here, it is not hard to understand the strong increase in household debt in ex-socialist countries over the past two decades. If households have strong consumption preferences towards the present and there are no central bank's restrictions, banks will meet the increase in demand for loans. This type of behavior was presented in Kraft and Jankov (2004).

Under the variable exchange rate, the choice of the exchange rate regime serves as a stopping tool for the increase in a household's debt. The variable exchange rate with the exchange rate risk transfer serves as a deterrent for households to get debt with the sole purpose of increasing their consumption.

Under the variable exchange rate policy, when the household obtains a loan in a foreign currency or with a foreign currency clause, it will have to compare its expectations of wage growth with those of the change in the exchange rate. If the household decides to increase its debt, it will have to compare the changes in the exchange rate with the changes in wages. If the exchange rate is increasing faster than wages, so will the annuity payments on loans.

The difference between hard and soft budget constraints is the source of imbalances in the economy. If households have a hard budget constraint, they will have to live within their budget constraint. If households do not have a hard budget constraint, but can borrow as much as they want from banks, the whole dynamics of the economic system of the small open economy changes, as observed in Zbašnik (2008).

With the ability to borrow freely, households can satiate their consumption as much as they want as long as banks are willing to provide the credit. The policy choice of a fixed exchange rate directly changes the behavior of households. However, the alternative behavior of households will change their relationship with other economic participants as well.

There is one major problem: households cannot live above their budget constraint for an infinite period of time. At one point in time, debt reaches the level where banks are no longer willing to give loans to the household. At that point, the household's consumption can be presented with the following equation:

$$I_t = w_t^e + \tau S_{t-1}^e - \kappa \Phi^*{}_{t-1} \tag{17}$$

Now the income of households is wages plus liquidated savings minus the repayment of debt. Debt is Φ* and this indicates that households have reached the upper limit of debt. Naturally, the paradox here is that if the household does not have any savings or it does not want to liquidate accumulated savings, disposable income for the household will be below wages. This hard landing will decrease the consumption of households. Even if the household decides to keep the unnaturally high level of consumption by liquidating savings, this strategy also has a limit since the savings households have accumulated are finite. The stop in lending and consequences of that stop have been described in Vidaković (2007a).

The choice of the variable exchange rate regime leads to a completely different outcome than the choice of the stable exchange rate regime. The variable exchange rate regime immediately serves as a hard budget constraint for households and a deterrent against living above their means.

2.1.3. Exchange rate regime switches

In this part, we will discuss the implications of the exchange rate regime change. We will model the switch in the monetary exchange rate regime and its implications. The focus will be on when the exchange rate regime changes from fixed to variable.

The switch from a stable to a variable exchange rate regime is more "stressful" for households since they have to learn the true model central banks are using to change the exchange rate. Because of the introduction of exchange rate risk, households will have to adjust their debt holdings. An increase in the exchange rate implies a decrease in consumption since depreciation causes debt annuity to go up. In addition, there will be an adjustment in obtaining new debt to finance consumption.

Once there is a change from a variable to a stable exchange rate regime, households switch their model as well. The change in the model is instantaneous. With the change in the model comes the change in the choices made. Households have to learn the model that the central bank is using. Models that use learning techniques and situations where expectations are not perfectly rational can be found in Hansen, Sargent, Turmuhambetova and Williams (2006), Marcet and Sargent (1989), Pearlman and Sargent (2005) and Woodford (2006).

When the central bank switches to a variable exchange rate, households have to learn the model the central bank is using to change the exchange rate. The model presented here closely follows that of Evans, Honkapohja, Williams and Sargent (2012). The rational expectations model for the movement in the exchange rate is:

$$e_t = \mu + \alpha E^*_{t-1} e_t + \delta z_{t-1} + \eta_t \tag{18}$$

where e is the nominal exchange rate, $E^*_{t-1}e_t$ denotes expectations of the nominal exchange rate on available information at t-1 and η_t is an unobservable white noise shock with $E\eta^2_t = \sigma^2_\eta$. The value of the intercept μ will be put to 0 for simplicity. The variable z is an exogenous observable variable following a stationary AR(1) process, which we will define as:

$$z_t = \rho z_{t-1} + w_t \tag{19}$$

where w_t is independently and identically distributed with distribution $(0, \sigma^2_t)$.

This particular setup gives us the ability to look at monetary policy from two separate perspectives. The first perspective is expectations E*, which do not have to be true rational expectations, but can be based on the subjective distributions of households on the future changes in the exchange rate. Expectations E* might be deeply rooted in a household's mentality, such as the fear of inflation or fear of devaluation as described in Gregurek and Vidaković (2008). In the stable rational expectations equilibrium, the subjective expectations operator E* should become objective rational expectations E* equal to the model the central bank has. The actual conduct of the central bank's monetary policy is given by exogenous shocks z. Since the household does not know what the central bank will do, they have to learn how the central bank works. From equations (18) and (19), the rational expectations solution is:

$$e_t = \frac{1}{(1-\alpha)}\delta z_{t-1} + \eta_t \tag{20}$$

For simplicity, we will set $\beta = (1-\alpha)^{-1}\delta$. What participants try to do is to learn the changes in β over time and to be able to do that they will have to employ Bayesian techniques. By using simplified notation, we can now ascertain participants' beliefs and understand how they evolve over time:

$$e_t = \beta z_{t-1} + \eta_t \tag{21}$$

where we will assume $\eta_t \perp z_{t-1}$ and $\eta_t \sim N(0, \sigma^2_\eta)$. Equation (21) is the foundation of the law of motion for β. As time goes by, exogenous shocks z will change the value of the variable β because households will learn the central bank's model. Therefore, now we will set the law of motion as:

$$e_t = b_{t-1} z_{t-1} + \eta_t \tag{22}$$

where b_{t-1} is the t-1 estimate of β. For mathematical purposes, we will assume the prior distribution of β is $\beta \sim N(b_0,V_0)$; the prior distribution implies a posterior distribution of $f(\beta \mid y^{t-1})$, where $y^t = (y_t, y_{t-1}, y_{t-2}, ...y_0)$ and $y'_t = (p_t, z_t)$ of the form $N(b_t,V_t)$. In order to update the parameters and get observations from which they can learn the model the central bank is using, households have to have a mechanism that allows them to learn through observations. The mathematical process of how to extract patterns from the data can be found in a method called the Kalman filter, based on Kalman (1960). We can define the Kalman filter as:

$$b_t = b_c + \frac{V_{t-1} z_{t-1}}{\sigma^2_\eta + V_{t-1} z^2_{t-1}}(e_t - b_{t-1} z_{t-1}) \tag{23}$$

$$V_t = V_{t-1} - \frac{z^2_{t-1} V^2_{t-1}}{\sigma^2_\eta + V_{t-1} z^2_{t-1}}(e_t - b_{t-1} z_{t-1}) \tag{24}$$

In Evans, Honkapohja, Williams and Sargent (2012) is formal proof the model converges with a probability 1 if $\alpha < 1$, and they also get that

$$V_t = \frac{\sigma^2_\eta}{(t-1)S_t - z^2_t} \to 0 \tag{25}$$

With a probability 1 for all σ^2_η regardless of whether σ^2_η is correct or not.

The model presented shows how households learn over time and how eventually the rational expectations model prevails: the central bank and economic participants end up having the same model.

We will now implement the learning process in our model and connect the household consumption choice with the learning model. We can define the variable exchange rate regime as:

$$f(e) = \sum_{t=0}^{n} (1 + \delta_t^e) x_0 \tag{26}$$

where δ is the change in the exchange rate given the actions performed by the central bank. The parameter δ is stochastic.

When the exchange rate regime changes, households are aware they have to modify their model of exchange rate risk. There are two possibilities: the rate of change in the exchange rate is known to households or the rate of change in the exchange rate is not known to households.

Case 1: the rate of change is known. In this case, expectations are made rationally and the Bellman equation has the same form as before:

$$V(\tau, s, \phi) = \max_{c} \left\{ u(c) - \beta E^{RE} V_{t+1}^{RE} (\tau_{t+1}, s_{t+1}, \phi_{t+1}, \Phi_{t+1}) \right\} \tag{27}$$

where RE now shows that expectations are made rationally under the rational expectations hypothesis.

Case 2: the rate of change in the exchange rate is not known to households. When the exchange rate regime switch is announced, the household does not know the average rate of depreciation and it has to make expectations using the probability distribution from equation (21).

Since the household does not know the conduct of monetary policy, it has to assume the probability distribution for the changes in the exchange rate. Initially, this distribution is wrong, but over time the household learns the true distribution by using the Kalman filter learning mechanism we described in equations (23) and (24). Because of the introduction of the Kalman filter, we have to change our transition equation to:

$$c_{t+1} = g(\tau_t, s_t, \phi_t, \theta_t) \tag{28}$$

where θ represents the expected value changes in the nominal exchange rate, based on the collected observations. The parameter θ is obtained from the Kalman filter through the learning process described in equations (23) and (24). Given this knowledge, we can change the Bellman equation to be:

$$V_t^B(\tau_t, s_t, \phi_t, \theta_t) = \max_{c} \left\{ f(u(c), \Phi) - \beta E^B V_{t+1}^B (A_{t+1}, \Phi_{t+1}, \theta_{t+1}) \right\} \tag{29}$$

The Bellman equation has now changed and it has Bayesian expectations noted with superscript B. The addition of the Bayesian expectations changes the whole process of the recursion. Under rational expectations, once the Bellman equation is obtained, households solve the dynamic programming problem and the solution is valid for every time period. With Bayesian expectations, that is not the case. Since the mean of the distribution of the exchange rate changes every period, the Bayesian household obtains the Bellman equation every period and then solves the dynamic programming in each time period, not just once as under rational expectations. From this process comes the aversion towards a high level of debt. Over time, the household obtains the correct distribution of the changes in the nominal exchange rate and the household and central bank will have the same distribution leading to the rational expectations equilibrium.

2.2. Banks

Banks are profit maximizing firms and will try to maximize profits under the given exchange rate regime. In eastern European countries, the role of banks in the economy and process of privatization has been sensitive. For example, Ribnikar (1995, 2004b) see banks as desirable to be privatized, but important enough for the economy not to be privatized fast. On the other hand, Kraft (2002, 2003) and Kraft, Faulend and Tepuš (1998) see the privatization of banks and sale of banks to foreigners as something very beneficial.

The central bank imposes monetary policy onto banks. Once banks know such monetary policy, they try to minimize the cost of regulation and maximize their profits. For the purposes of this paper, it is important to understand the portfolio choices of banks under different exchange rate regimes and the difference, if any, in the sector distribution of credit.

We will model banks as utility maximizing economic agents. Banks behave in the same way as households do, except the object of the maximization is not consumption, but profits. In banking, profit comes from buying money (getting deposits from primary and/or secondary sources of funds) at some price and selling it (giving out loans or participating in trading activities) at a higher price. The difference between the cost of funds and the price of funds "sold" provides a bank's net interest income and consequently profit. The pursuit of profit takes place while banks are trying to solve the problem of minimizing business risks and maximizing profits.

The assumption of maximizing profits, while minimizing risk, is the theoretical basis for using the utility function. We will assume the utility function has the following standard properties $u'(\bullet)>0$ and $u''(\bullet)<0$, the utility function is continuous and at least twice differentiable. The increase in profit is necessarily tied to the increase in risks taken. The bank wants more profit; however, an increase in profits is followed by an increase in risk, making the pleasure of each new dollar earned under higher risk less and less pleasurable. We will assume the bank has the option to invest in as many investments as it can get funding for and that investments have a positive rate of return.

Profit is an accounting variable, not an economic variable. In accounting terms, profit is the difference between income and cost. The bank can influence either income or cost, and then see the results in profits. The profit can be influenced only indirectly. In order to have profit in the utility function, we have to derive the connection between income and expenditure. The formula for profit will be:

$$\pi = rA - \delta L \tag{30}$$

where π is profit, A is a matrix for assets, L is a matrix for liabilities[4], r is the vector of the interest rates on assets and δ is the vector of the interest rates on liabilities. Assets have to equal liabilities. Because of funding, we get the following equation for a bank's profit:

[4] We will only look at interest bearing assets and interest bearing liabilities plus capital. We will not look at other items on the bank's balance sheet. Therefore, when we refer to assets or liabilities, we are referring to the interest bearing parts of the balance sheet plus capital.

$$\pi = rA - \delta A = A(r - \delta) = \tau A \tag{31}$$

where τ is the vector of the net effective interest rate the bank gets or the interest rate spread between assets and liabilities. We assume there are no other costs. Although this is not technically correct, we are more focused on the decisions banks make, not on the actual value of profits.

We will use a CRRA class of utility function so the bank has a relative risk per unit of exposure. The bank has a fixed percentage of risk acceptances for each investment. This gives the bank flexibility in its investments, but at the same time it has a fixed risk tolerance. The utility function is:

$$u(\pi) = \frac{\pi^{1-\gamma}}{1-\gamma} \tag{32}$$

where π is profit and γ is the elasticity of substitution with the value $0<\gamma<1$. The importance of the choice of utility function can best be found in Kimball (1993). The value of the choice of utility function lies in the standard risk aversion measure proposed by Kimball (1993):

$$-\frac{u''}{u'} \tag{33}$$

Banks face two separate optimization problems. The first problem is how to maximize profits from the credit portfolio, which is derived from the funds collected in liabilities and then allocated in assets. The second optimization problem is how to minimize the cost of regulation. The assets of the bank will be separated into two vectors, shown in the matrix form they are

$$A_t = \begin{bmatrix} x_t \\ q_t \end{bmatrix} \tag{34}$$

where A_t is total assets at time t, x is the interest bearing assets and q is the part of assets allocated specifically as demanded by the regulation. Since we are not interested in the interplay between the central bank and bank, we will only focus on the x part of the balance sheet and treat q as fixed.

The problem will be stochastic since banks face risk in their business and have to create expectations about the future. The basic setup of the problem can be found in Cooper and Adda (2003). The bank tries to maximize the present value of expected utility from profits over time:

$$\max \ E\left[\sum_{t=0}^{\infty} \beta^t u(\pi_t)\right] \tag{35}$$

Profit is noted as π, discounted by the rate β in order to get the present value of profit since β has values $0<\beta<1$, while E stands for the expectations operator. The value function is:

$$V_t(x_t) = \max \; E\left[\sum_{t=0}^{\infty} \beta^{-t} u(\pi_t)\right] \tag{36}$$

Subject to $x_0 > 0$, $x_\infty > 0$ is free; $x_t > 0$ for all time periods. We assume x has some initial value; the end value is free, so we do not impose a growth limit on banks. We also assume that over time the value of x is greater than 0, since it would be impossible for the bank to have no interest bearing assets in this part of the balance sheet. The transition equation for the interest bearing part of the balance sheet is:

$$x_t = x_{t-1} + (\Lambda_t - \lambda_t) \tag{37}$$

In each period, the bank has net interest bearing assets x_{t-1} from the previous period and the current period changes in the values of interest bearing assets, where Λ_t represents the incoming funds into the bank, while λ_t are the outgoing funds in each time period. Note that we do not use savings or the repayment of credit as inflows or new debt as outflows of funds from the bank. This is because the bank's domestic funding might not be enough to meet the demand for loans, and so we have to broaden the categories and just define inflows and outflows from all possible sources.

Since we are investigating whether banks alter their lending under alternative exchange rate regimes, we will give banks two possibilities: to lend to households and to lend to all other participants in the economy. The formula for profit in each time period will be:

$$\pi_t = [x_{t-1} + (\Lambda_t - \lambda_t)][zr_t\omega_t + r_t^e(1 - \omega_t)] \tag{38}$$

In equation (38), we see two rates of return and two classes of assets. The first rate of return is z, which is the average rate of return on all other assets in the balance sheet and it is the portion of ω of the interest bearing assets. The second group is loans to households; those loans have the 1-ω portion of the risk bearing assets and the expected rate of return is r. As we can see, both rates have subscript t, which denotes the time period, and superscript e, which denotes expectations about the risk bearing rate of return for banks. Now we can set up the Bellman equation:

$$V_t(x_t) = \max_{\pi_t, \omega_t} \left\{u(\pi) + \beta^{-1} E\left[V_{t+1}(x_{t+1})\right]\right\} \tag{39}$$

The E in the Bellman equation indicates expectations, since the bank has risk bearing assets where the rate is not known with certainty, but it has to be obtained through expectations.

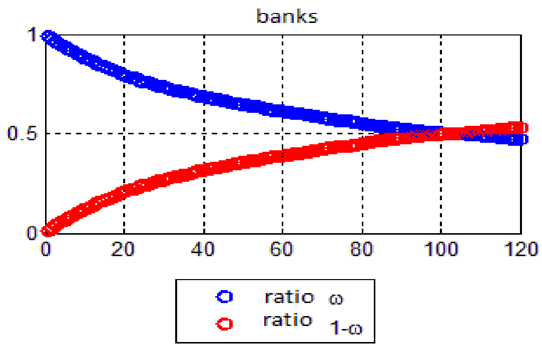
3. Model simulation

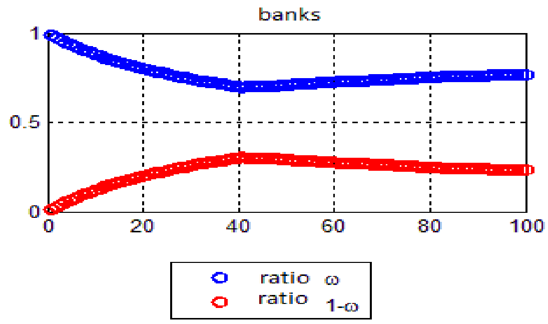
We will test the model in two separate ways. First, we will perform a Matlab simulation of the model and in the next part of the paper, we will perform a real life data test.

The parameters used for the model are given in the appendix. Going back to the main thesis of the paper, we should see different amounts of credit that households are willing to hold under different exchange rate regimes. Under a variable exchange rate regime, we should see smaller amounts of credit because households do not prefer large debt holding because of foreign exchange risk. On the other hand, under a stable exchange rate, we should see higher debt holding.

In the model, we start with ω=1, and so there is no household debt. Picture 1 shows what happens under a stable exchange rate regime and Picture 2 shows what happens under a variable exchange rate regime.

Picture 1. : ω under a stable exchange rate regime





Picture 2. : ω under a variable exchange rate regime

Source: author's calculation

Under both exchange rate regimes, we see an increase in the value of ω, meaning that the household's debt starts to increase. However, there is one main difference between stable and variable exchange rate regimes. Under a variable exchange rate regime, lending to households peaks at around 30% of total lending and then starts to decrease. On the other hand, lending to households under a stable exchange rate regime steadily increases over time and, as we can see close to the end of the simulation, it takes over as the majority of a bank's lending. From the simulation, we can see that under a stable exchange rate regime, over time lending to households completely crowds out lending to other participants in the economy.

4. Real life example

The real life example will be performed on Croatia, Hungary, Latvia and Slovakia. First, we have to define exchange rate regimes. As is obvious from the data in Table 1, there is no need for econometric testing for Hungary and Slovakia since from the data we can see the exchange rate is variable and then it switches to stable. Therefore, the econometric testing will be for only Croatia and Latvia. The test used is the Chow test based on Chow (1960), which is used to determine structural breaks in the data. In our case, the cause for the structural break is the change in the exchange rate regime.

In the case of Croatia, we will use the ARMA regression. By using the monthly time series for the exchange rate, we will have the following regression:

$$EX = \alpha + \beta_1 EX_{t-1} + \beta_2 EXMA_{12} \quad (40)$$

where EX is the exchange rate and $EXAM_{12}$ is the 12-month moving average of the exchange rate. The two time periods we split the data into are 1994–2000 and 2001–2008 for Croatia. By using the F distribution table for k=3 and 158, we find that the p value is 4.8%, which is just below the 5% significance level. In this particular case, we will reject the null hypothesis and state there was a regime switch in Croatia.

For Latvia, the first time period will be 1993–1999 and the second time period will be 2000–2008. The F value we will base on the F distribution with k and t-2k. In our case, that is 3 and 186. The p value we have obtained is 0 to the 4th decimal clearly indicating that there was a structural break.

From Table 1, we can see the data confirmation of the model. In Hungary, the data follow the model perfectly. Household loans start with almost 40% at the start of the data and we can clearly see the decrease in the amount of household loans. Then in 2001, Hungary stabilizes the currency and loans to households start growing (the amount of loans to households as a proportion of total loans more than doubles). With the start of the financial crisis in 2008, the Hungarian Forint starts to depreciate and the portion of loans stabilizes (from 2009 to 2011, it is almost unchanged). In the same time period, the currency also depreciates by 15%, meaning that a portion of the growth of loans can also contribute to the depreciation of the exchange rate, not to the actual growth of loans.

Table 1: ω under a variable exchange rate regime

	Croatia		Hungary		Latvia		Slovakia	
end of year	Croatian kuna	1-ω	Hungarian forint	1-ω	Latvian Lats	1-ω	Slovečka Krona	1-ω
1990	:		83,77	39,7%	:			
1991	:		101,40	27,8%	:			
1992	:		101,68	29,2%	1,01			
1993	:	9,4%	112,35	30,1%	0,67		37,04	7,3%
1994	6,96	12,7%	136,73	26,1%	0,67		38,48	6,5%
1995	6,96	14,5%	183,30	21,3%	0,71		38,86	5,4%
1996	6,94	19,6%	206,91	14,9%	0,70		39,95	5,0%
1997	6,96	26,3%	224,71	16,1%	0,66	2,4%	38,43	6,1%
1998	7,29	29,7%	252,39	14,1%	0,66	5,2%	43,21	7,6%
1999	7,68	34,7%	254,70	15,0%	0,59	13,3%	42,40	10,1%
2000	7,58	38,6%	265,00	16,8%	0,58	17,3%	43,93	12,1%
2001	7,37	40,5%	245,18	21,9%	0,56	21,8%	42,78	18,3%
2002	7,48	44,8%	236,29	28,9%	0,61	26,7%	30,13	21,3%
2003	7,65	49,8%	262,50	36,1%	0,67	36,0%	30,13	25,5%
2004	7,67	51,9%	245,97	40,5%	0,70	42,4%	30,13	35,1%
2005	7,37	53,3%	252,87	42,2%	0,70	50,0%	30,13	40,9%
2006	7,35	52,9%	251,77	42,3%	0,70	59,1%	30,13	42,0%
2007	7,33	54,5%	253,73	43,8%	0,70	61,1%	30,13	42,9%
2008	7,32	54,7%	266,70	46,1%	0,71	61,4%	30,13	44,9%
2009	7,31	53,5%	270,42	48,1%	0,71	62,0%	1,00	48,3%
2010	7,39	52,3%	277,95	50,8%	0,71	61,9%	1,00	50,8%
2011	7,53	50,0%	311,13	50,2%	0,70	60,2%	1,00	51,5%

Source: author's calculation and central bank's data

A similar pattern can be seen in Slovakia as well. The ratio of loans in the bank's balance sheet was low as long as the currency was depreciating. Then in 1998, the currency stabilizes and loans to households start to grow as a proportion of total loans. The increase is even faster as Slovakia stabilizes the currency because of joining the EMU.

What is most startling is the crowding out effect loans to households have on other loans. Just like the case of our model, we see loans to households over time take over as the majority of loans on the balance sheet.

The model is consistent for Latvia as well. The ratio of a household's loans is increasing, which we see in the model, but it truly explodes once the exchange rate stabilizes.

5. Conclusion

This paper investigates the effects of the exchange rate regime choice on banks, households and distribution of debt. The main feature of the model presented in this paper is the ability of households to hold both debt and savings at the same time. In this property, the model significantly differs from other models where savings equal loans. In this model, savings can equal credit and credit can be used to increase consumption. Further, the model assumes the FX risk plays a significant role in the actual decision-making process of households. If the exchange rate is stable, the propensity of households towards debt is greater. On the other hand, if the exchange rate regime is variable, the very existence of the FX risk will serve as a deterrent towards debt holding by households.
The model is tested in both laboratory and real life settings. We can see two main facts from the tests: the choice of the exchange rate regime plays a significant role in terms of the debt holdings of households and the savings equal loans assumption needs to be revised when modeling eastern European transition economies.
Apart from the confirmation of the model in the data, another implication of the model is the importance of the exchange rate regime on the bank's credit policy. This paper shows that just the choice of the exchange rate regime has a significant impact on the economy.

Appendix 1: the data

In order to calibrate the model for the Matlab simulation, we used the following data and parameterization. Wages were taken from Croatia in period 31.12.2001 – 31.12.2008. For the variable exchange rate, the Slovenian exchange rate was used from period 10.1999 – 12.2003. Savings were set as 20% of wages. This assumption is empirically consistent with Croatia.
Initially, the banks' balance sheet was set at 0 loans to households. Interest rates on loans and deposits were used from the Croatian central bank's web site. For the utility functions, the parameters γ and θ were put at 2. The discount factor β is 4% per year. All these factors are consistent with the parameters used in Bokan, Grguric, Krznar and Lang (2009).